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More than hundred (100) estimators for estimating the shrinkage parameter in a linear and generalized linear ridge regression models

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Abstract

This paper reviewed many papers and provided more than 100 different available estimators for estimating the ridge or shrinkage parameter k for the Gaussian linear regression model. These estimators may be used for generalized linear regression models, namely, Poisson regression, Logistic regression, Beta regression, Gamma regression, zero inflated Poisson regression, negative binomial (NB) regression, zero inflated NB, Bell regression and inverse Gaussian regression models among others. It is expected that this paper will bring a lot of attention among the researchers and will be used as a reference paper in the area of ridge regression, which is mainly used to solve the multicollinearity problems.

Keywords: Generalized Linear Regression; Linear Regression; OLS; MSE; Multicollinearity; Ridge Regression; Shrinkage parameters.

1. Introduction

Multiple linear regression model plays an essential role in statistical inference and is used extensively in business, environmental, industrial, medical, and social sciences. In linear regression model, one usually assumes that the explanatory (or regressors) variables are independent. However, in practice, there may be strong or near to strong linear relationships among the explanatory variables. In that case the independence assumptions are no longer valid, which causes the problem of multicollinearity. In the presence of multicollinearity, it is difficult to estimate the unique effects of individual variables in the regression equation. Moreover, the estimated regression coefficient may have wrong sign and will have unduly large sampling variance, which affects both inference and prediction (Hoerl and Kennard, 1970). In literature, there are various methods exist to solve the multicollinearity problem. Among them, "ridge regression" proposed by Hoerl and Kennard (1970) is the most popular one which has much usefulness in real life. They suggested the ridge regression (RR) as an alternative approach to OLS. To apply RR, a researcher needs to determine the value of ridge or shrinkage parameter k. Since 1970, the researchers are focusing on the estimation of k using different methods and various conditions and then compare their results with the OLS estimator. There are many research have been done for the estimation of ridge parameter or shrinkage parameter k for the linear regression model. More on ridge regression model and estimation of shrinkage parameter k, interested readers are refer to Hoerl and Kennard (1970), Marquardt (1970), Theobald (1974), Hawkins (1975), Hoerl et al. (1975), McDonald and Galarneau (1975), Lawless and Wang

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(1976), Hocking et al. (1976), Wahba et al. (1979), Gibbons (1981), Schaefer et al. (1984), Delaney and Chaterjee (1986), Nomura (1988), Crouse et al. (1995), Firinguetti (1999), Kibria (2003), Khalaf and Shukur (2005), Alkhamisi et al. (2006), Alkhamisi and Shukur (2006, 2007), Batah et al. (2008), Muniz and Kibria (2009), Dorugade and Kashid (2010), Hassan (2010), Massson et al. (2010), Muniz et al. (2012), Khalaf (2013), Khalaf et al. (2013), Asar et al. (2014), Khalaf and Iguernane (2014), Dorugade (2014), Göktaş and Sevinç (2016), Khalaf and Iguern (2016), Bhat and Vidya(2016), Karaibrahimoglu, *et al.* (2016), Dorugade (2016), Lukman et al. (2017), Asar and Genc (2017), Lukman and Olatunji (2018), Ertas (2018), Suhail and Chand (2019), Suhail et al. (2019), Ali et al. (2021), Zubair and Adenomon (2021), Dar et al. (2022), Karakoca (2022), Khalf (2022), and references therein. Below, we will provide a brief descriptions about some generalized linear regression models.

Since, the linear regression model (LRM) is most popular and has been used extensively than any other models, a lot of attention has been given to the work on the LRM in which the response variable has the normal distribution. However, in reality, there are the situations, where the data often comes from the other exponential family of distributions such as gamma, Poisson, negative binomial, exponential, logistic, bell, inverse Gaussian and others instead of the normal distribution. In such situations, the generalized linear model (GLM) is the best choice instead of the linear regression model. When the dependent variable of the regression model is positively skewed and mean is proportional to dispersion parameter, one uses the gamma regression model (GRM). For generalized linear ridge regression model, we refer Segerstedt (1992) and for the gamma ridge regression models, we refer to Lukman et al (2020, 2021), Amin et al. (2020a, 2020b), Amin et al (2022), Yasin et al. (2022), Lukman et al. (2022) and Akram et al. (2022) and references therein.

The beta regression model (BRM) is introduced by Ferrari and Cribari-Neto (2004), which often used when the dependent variable is in the form of rates or proportion. The primary assumption of the BRM is that the dependent variable is distributed as a beta distribution. For instance, when modelling the proportion of income spent on food, the poverty rate, the proportion of death from Covid-19, and the proportion of surface covered by vegetation. In such situations, the BRM has been used in applied research. The maximum likelihood estimator (MLE) is used to estimate the unknown regression coefficient of the BRM. More on Beta ridge regression model, we refer Qasim et al. (2021), Akram et al (2021) and Abonazel and Taha (2021) references therein.

Logistic regression model is a popular method to model binary data**.** This model (also known as *logit model*) is frequently used for classification and predictive analytics. Logistic regression estimates the probability of an event occurring, such as cure or not cure a disease, based on a given dataset of independent variables. Since the independent variables may be correlated, the ridge regression method can be used for Logistic regression model. More on logistic regression and logistic ridge regression model, we refer our readers to Schaefer et al. (1984), Cessie and Houwelingen (1991), Kibria et al. (2012), Mansson et al. (2014), William et al. (2019) among others.

Count data regression is more appropriate than the linear regression model in studying the occurrence rate per unit of time conditional on some covariates. For examples, the number of patents visit emergency room, takeover bids, bank failures, and the number of accidents on a highway. Unless the mean of the counts is high, using the LSE can lead to significant deficiencies. In such situations, the model for count data is the Poisson regression model. When covariate for Poisson regression models are correlated, a Poisson ridge regression model is preferred. There are much research on Poisson ridge regression model. Some of the notable are Mansson and Ghazi (2011b), Kibria et al. (2015), KaÇiranlar and Dawoud (2018), Qasim et al. (2020), Lukman et al. (2021), Omer et al. (2021) and references therein.

The zero-inflated Poisson (ZIP) regression model introduced by Lambert (1992) is a popular choice among researchers in applied economics when the dependent variable comes in the form of non-negative integers or counts. The ZIP model usually used when the data contain an excess amount of zeros. It is popular model because it accounts for overdispersion. For zero inflated ridge regression estimator, we refer to Kibria et al. (2013), Omer (2021) and very recently Al-Taweel and Algamal (2022) and references therein.

The important assumption of the Poisson regression model is that the conditional mean and variance of the dependent or outcome variable are equal. In real life applications, the conditional variance may exceed the conditional mean, which is commonly referred to as overdispersion. Then the Poisson regression model should be replaced by the negative binomial (NB) regression model, since NB regression incorporates an additional term to account for the excess variance. More on negative Binomial regression and ridge regression models, we refer to Mansson (2012), KaÇiranlar and Dawoud (2018), Alobaidi et al. (2021), and very recently Rashad et al. (2021) and references therein.

In some situations, a huge number of zeros may be included in the count data. In that situation, the Zero-inflated negative binomial regression (ZINB) models are commonly used for count data that show overdispersion and extra zeros. More on zero-inflated negative binomial model and zero-inflated negative binomial ridge regression model, we refer to Cameron and Trivedi (2013), Preisser (2016), Al-Taweel and Algamal (2020) and very recently Akram et al. (2022) references therein.

The inverse Gaussian regression (IGR) model is a well-known model in the application when the response variable positively skewed. More on inverse Gaussian regression model we refer Folks et al. (1981), Bhattacharyya and Fries (1982), Chaubey (2002) and Heinzl and Mittlbock (2002) among others. However, for the ridge regression model under the inverse Gaussian model is consider by Algamal (2019), Amin et a. (2021a) and very recently Amin et al. (2021) and references therein.

The Bell regression model (BRM) is generally applied in a situations, when the response variable having observed counts that follows the Bell distribution (Lemonte at al. 2020). More on the Bell ridge regression model, we refer our readers to Amin et al. (2021b) and references therein.

We have reviewed several regression models, namely, Linear regression model, Gamma regression model, Poisson regression model, Logistic regression model, Zero inflated Poisson regression model, Negative binomial regression model, Zero inflated NB regression model, Bell and finally Inrese Gaussian ridge regression models. However, in this paper we will consider the case for linear regression model only. Since, the estimation of the ridge or shrinkage parameter is an important issue for the ridge regression model, the objective of the paper is to review some papers since Hoerl and Kennar (1970) and provided at least 100 different k estimators. The organization of this paper is as follows: Statistical models are outlined in section 2. 108 different ridge parameters (between 1970 to 2022) are given in section 3. This paper ends up with some concluding remarks in section 4.

2. Statistical Models

To describe the ridge regression models, we consider the following multiple linear regression model

$$y = X\beta + e, \tag{2.1}$$

where y is an nx1 response vector of observations on the dependent variable, X be the (nxp) design matrix of rank p, β is an (px1) vector of unknown parameters to be estimated, and e an (nx1) vector of unobservable errors with 0 mean and variance σ^2. The regression parameters vector, β is mostly estimated using the method of Least Squares (LS) when there is no violation of any of the classical linear regression Model (CLRM) assumptions (Hoerl and Kennard 1970). The ordinary least squares (OLS) of β is defined as follows:

$$\hat{\beta}_{LS} = (X'X)^{-1}X'y \tag{2.2}$$

with covariance matrix, $Cov(\hat{\beta}) = \sigma^2(X'X)^{-1}$. It can be seen that both $\hat{\beta}$ and $Cov(\hat{\beta})$ are heavily dependent on characteristics of the matrix X'X.

The OLS estimator is an unbiased and has minimum variance among the class of all such unbiased linear estimator. In a multiple linear regression model, it is generally assumed that predictors must be uncorrelated with each other. However, in many practical situations (e.g. engineering in particular (Hoerl and Kennard, 1970)), often find that the regressors are nearly dependent. In that case $X'X$ matrix becomes ill conditioned (i.e. $\det(X'X) \approx 0$). If $X'X$ is ill conditioned, then $\hat{\beta}$ is sensitive to a number of errors i.e. regression coefficients may have wrong signs with large, the sampling variance. In this situation the meaningful statistical inference about the regression coefficients becomes very difficult for practitioners. To overcome this problem, (Hoerl and Kennard 1970) proposed ridge regression estimator (RRE) which can be obtained by augmenting equation (2.1) with $0 = k^{1/2}\beta + \varepsilon$ and consequently applying the method of least squares to estimate β. Thus, the ridge estimator of β is obtained as:

$$\hat{\beta}_k = (X'X + kI)^{-1}X'y \quad ; k \geq 0 \tag{2.5}$$

where k is the shrinkage or biasing parameter. This is known as the ridge regression estimator (RRE). The RRE provides biased but smaller variance than the OLS estimator (Hoerl and Kennard 1970). From (2.5), we observe that as $k \to 0$, $\hat{\beta}_k \to \hat{\beta}$, and as $k \to \infty$, $\hat{\beta}_k \to 0$.

Now, we consider the canonical form of model (2.1) as follows

$$Y = X^{*}\alpha + \varepsilon, \tag{2.6}$$

where $X^{*} = XP$ and $\alpha = (\alpha_1, \alpha_2, \alpha_3 \dots, \alpha_p) = P'\beta$, P is an orthogonal matrix such that $P'P = I_p$ and $P'X'XP = \Lambda$, where $\Lambda = diag\ (\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_p)$ is the diagonal matrix consisting of the eigen values of $X'X$. Then, the ordinary least square (OLS) and the generalized ridge regression (GRR) estimators in canonical form are respectively given as follows:

$$\hat{\alpha}_{LS} = (X^{*\prime}X^{*})^{-1}\, X^{*\prime}Y, \tag{2.7}$$
$$\hat{\alpha}_{RR} = (X^{*\prime}X^{*} + K)^{-1}X^{*\prime}Y, \tag{2.8}$$

where $K = diag(k_1, \dots, k_p), k_i > 0; i = 1,2, \dots, p.$

The MSE of OLS and generalized GRR estimator in terms of eigen values and ridge parameter k can be written respectively as follows:

$$MSE(\hat{\alpha}_{LS}) = \sigma^2 \sum_{i=1}^{p} \frac{1}{\lambda_i}, \tag{2.9}$$
$$MSE(\hat{\alpha}_{RR}) = \sigma^2 \sum_{u=1}^{p} \frac{\lambda_i}{(\lambda_i + k_i)^2} + \sum_{u=1}^{p} \frac{{k_i}^2\alpha_i^2}{(\lambda_i + k_i)^2}. \tag{2.10}$$

The first term on the right of (2.10) is the variance and second term is the square of the bias introduced by the ridge regression estimator (RRE). This second term will be zero when *k=0*, however, it is a monotonic increasing function of *k*. On the other hand, the variance is a monotonic decreasing function of *k*. Thus, when *k* increases, the variance decrease and bias increase. Hoerl and Kennard (1970) shows that there always exists a value of *k>0*, for which MSE of RRE is smaller than the mean square error of OLS estimator.

3. Different Shrinkage Estimators

The parameter k is known as the “biased” or “ridge” or “shrinkage” parameter and it must be estimated using real data. Most of recent efforts in the area of multicollinearity and ridge regression estimators have concentrated on estimating value of *k*. We will review many statistical methodology used to analyze the estimation of *k* in this section.

For the linear regression model, Hoerl and Kennard (1970) obtained the optimal values of k_i as the ratio of estimated error variance ($\hat{\sigma}^2$) and i^{th}estimate of α using OLS as follows:

$$\hat{k}_i = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}, \quad ; i = 1,2,3, \dots, p \tag{2.11}$$

where $\hat{\sigma}^2 = \sum_{j=1}^{n} \hat{e}_j / (n-p)$.

To determine a single value for k, Hoerl and Kennard (1970) suggested the following estimator:

$$\hat{k}_{HK} = \frac{\hat{\sigma}^2}{\hat{\alpha}_{max}^2} \tag{2.12}$$

where $\hat{\alpha}_{max} = max(\alpha_1, \alpha_2, \alpha_3 \dots, \alpha_p)$. The estimator in (2.12) will give smaller MSEs than the OLS estimator. Hoerl et al. (1975) consider another estimator by taking the harmonic mean of k_i values in (2011). Kibria (2003) proposed estimators of k by using the arithmetic, geometric means and median of k_i values in (2.11). This paper follows the notation of Hoerl and Kennard (1970). Different researchers estimated the value of k in different ways. In this section, we will summarize some existing methods of estimating ridge parameter k and their corresponding references which are provided in Table 2.1.

Table 2.1: Different authors and their corresponding proposed ridge or shrinkage estimators

SL No	Authors	Estimators
1.	Hoerl and Kennard (1970)	$k_1 = \frac{\hat{\sigma}^2}{\hat{\alpha}_{max}^2}$
2.	Theobald (1974)	$k_2 = \frac{2\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}}$
3.	Hoerl at al. (1975)	$k_3 = \frac{p\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\alpha_i^2}$
4.	McDonald and Galarneau (1975).	Choose a value of k, (say k_4), such that $\hat{\beta}_k{}'\hat{\beta}_k = \hat{\beta}'\hat{\beta} - \hat{\sigma}^2 \sum_{i=1}^{p} \frac{1}{\lambda_i}$
5.	Lawless and Wang (1976)	$k_5 = \frac{p\hat{\sigma}^2}{\hat{\alpha}'X'X\hat{\alpha}} == \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\lambda_i\alpha_i^2}$
6.	Hocking et al. (1976)	$k_6 = \frac{\hat{\sigma}^2 \sum_{i=1}^{p}(\lambda_i\hat{\alpha}_i)^2}{\left(\sum_{i=1}^{p}\lambda_i\hat{\alpha}_i\right)^2}$
7.	Golub, Heath and Wahba (1979)	$k_7 = \frac{y'[I - H(k)]y}{n[Det(I_n - H(k))]^{1/2}}$ where $H(k) = X\left(X^T X + kI_p\right)X^T$
8.	Schaefer et al. (1984)	$k_8 = \frac{1}{\hat{\alpha}_{max}^2}$
9.	Delaney and Chatterjee (1986)	$k_9 = min\left(MSEP(k_g)\right)$, where $MSEP\left(k_g\right) = \frac{\sum_{i=1}^{p} MSEP_i(k_g)(U_i)}{\sum_{i=1}^{p} U_i}$ For detailed see Delaney and Chatterjee (1986).
10.	Nomura (1988)	$k_{10} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\left[\frac{a_i^2}{1 + (1 + \lambda_i\left(\frac{a_i^2}{\sigma^2}\right)^{1/2}}\right]}$
11.	Crouse et al. (1995),	$k_{11} = \begin{cases} \frac{p\hat{\sigma}^2}{a - b} & if\ a > b \\ \frac{p\hat{\sigma}^2}{a} & otherwise \end{cases}$ where $a = \left(\hat{\beta} - J\right)'\left(\hat{\beta} - J\right)$, $b = \hat{\sigma}^2 tr(X'X)^{-1}$ and $J = \left(\frac{1}{p}\sum_{i=1}^{p}\hat{\beta}_i\right) I_p$
12.	Firinguetti (1999)	$k_{12} = \frac{\lambda_i\,\hat{\sigma}^2}{((n - p - 1)\hat{\sigma}^2 + \lambda_i\,\hat{\alpha}_i^2)}$
13.	Kibria (2003)	$k_{13} = \frac{\hat{\sigma}^2}{\left(\prod_{i=1}^{p}\hat{\alpha}_i^2\right)^{1/p}}$
14.	Kibria (2003)	$k_{14} = median\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right)$

15.	Kibria (2003)	$k_{15} = \frac{1}{p}\sum_{i=1}^{p}\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$
16.	Khalaf and Shukur (2005)	$k_{16} = \frac{\lambda_{max}\,\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\,\hat{\beta}_{max}^2)}$
17.	Alkhamisi et al. (2006)	$k_{17} = max\left(\frac{\lambda_i\,\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_i\,\hat{\beta}_i^2)}\right)$
18.	Alkhamisi et al. (2006)	$k_{18} = median\left(\frac{\lambda_i\,\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_i\,\hat{\beta}_i^2)}\right)$
19.	Alkhamisi et al. (2006)	$k_{19} = \frac{1}{p}\sum_{i=1}^{p}\left(\frac{\lambda_i\,\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_i\,\hat{\beta}_i^2)}\right)$
20.	Alkhamisi and Shukur (2007)	$k_{20} = \frac{\hat{\sigma}^2}{\hat{\beta}_{max}^2} + \frac{1}{\lambda_{max}}$
21.	Alkhamisi and Shukur (2007)	$k_{21} = max\left(\frac{\hat{\sigma}^2}{\hat{\beta}_i^2} + \frac{1}{\lambda_i}\right)$
22.	Alkhamisi and Shukur (2007)	$k_{22} = \frac{1}{p}\sum_{i=1}^{p}\left(\frac{\hat{\sigma}^2}{\hat{\beta}_i^2} + \frac{1}{\lambda_i}\right)$
23.	Alkhamisi and Shukur (2007)	$k_{23} = Median\left(\frac{\hat{\sigma}^2}{\hat{\beta}_i^2} + \frac{1}{\lambda_i}\right)$
24.	Alkhamisi and Shukur (2007)	$k_{24} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\hat{\beta}_i^2} + \frac{1}{\lambda_{max}}$
25.	Alkhamisi and Shukur (2007)	$k_{25} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\lambda_i\hat{\beta}_i^2} + \frac{1}{\lambda_{max}}$
26.	Batah et al. (2008)	$k_{26} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\left[\frac{\hat{\alpha}_i^2}{1+(1+\lambda_i(\hat{\alpha}_i^2/\hat{\sigma}^2))^{1/2})}\right]}$
27.	Muniz and Kibria (2009)	$k_{27} = max\left(\frac{1}{\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}}\right)$
28.	Muniz and Kibria (2009)	$k_{28} = \max\left(\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}\right)$
29.	Muniz and Kibria (2009)	$k_{29} = \left(\prod_{i=1}^{p}\frac{1}{\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}}\right)^{1/p}$
30.	Muniz and Kibria (2009)	$k_{30} = \left(\prod_{i=1}^{p}\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right)^{1/p}$

31.	Muniz and Kibria (2009)	$k_{31} = median\left(\frac{1}{\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}}\right)$
32.	Muniz and Kibria (2009)	$k_{32} = median\left(\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}\right)$
33.	Muniz and Kibria (2009)	$k_{33} = \left(\prod_{i=1}^{p} \frac{\lambda_i\,\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_i\hat{\alpha}_i^2}\right)^{1/p}$
34.	Batah and Gore (2009)	$k_{34} = \frac{p\,\hat{\sigma}^2}{\sum_{i=1}^{p} \frac{\hat{\alpha}_i{}^2}{\left(\frac{\hat{\alpha}_i{}^4\lambda_i{}^2}{4\hat{\sigma}^4} + \frac{6\hat{\alpha}_i{}^2\,\lambda_i}{\hat{\sigma}^2}\right)^{1/2} - \frac{\lambda_i\hat{\alpha}_i{}^2}{2\hat{\sigma}^2}}}$
35.	Dorugade and Kashid (2010)	$k_{35} = max\left(0, \frac{p\hat{\sigma}^2}{\hat{\alpha}'\hat{\alpha}} - \frac{1}{n\left(VIF_j\right)_{max}}\right),$ $VIF_j = \frac{1}{1-R_j^2}$ is the variance inflation factor of jth regressor.
36.	Al Hassan (2010)	$k_{36} = \frac{\sum_{i=1}^{p}(\lambda_i\hat{\alpha}_i)^2}{\left(\sum_{i=1}^{p}\left(\lambda_i\hat{\alpha}_i{}^2\right)\right)^2} + \frac{1}{\lambda_{max}}$
37.	Mansson, Shukur and Kibria (2010)	$k_{37} = max\left(\frac{1}{\frac{\lambda_{max}\,\hat{\alpha}_i^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\,\hat{\alpha}_i^2)}}\right)$
38.	Mansson, Shukur and Kibria (2010)	$k_{38} = median\left(\frac{1}{\frac{\lambda_{max}\,\hat{\alpha}_i^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\,\hat{\alpha}_i^2)}}\right)$
39.	Muniz et al. (2012)	$k_{39} = \left(\prod_{i=1}^{p} \frac{\lambda_i\,\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_i\,\hat{\alpha}_i^2)}\right)^{1/p}$
40.	Muniz et al. (2012)	$k_{40} = max\left(\frac{1}{\sqrt{\frac{\lambda_{max}\,\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\,\hat{\alpha}_i^2)}}}\right)$
41.	Muniz et al. (2012)	$k_{41} = max\left(\sqrt{\frac{\lambda_{max}\,\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\,\hat{\alpha}_i^2)}}\right)$

42.	Muniz et al. (2012)	$k_{42} = \left(\prod_{i=1}^{p} \frac{1}{\sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_i^2)}}} \right)^{1/p}$
43.	Muniz et al. (2012)	$k_{43} = \left(\prod_{i=1}^{p} \sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_i^2)}} \right)^{1/p}$
44.	Muniz et al. (2012)	$k_{44} = median \left(\frac{1}{\sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_i^2)}}} \right)$
45.	Khalaf (2013)	$k_{45} = \frac{(\lambda_{max} + \lambda_{min})}{2\sum_{i=1}^{p}\lvert\hat{\beta}\rvert_j} \left\{ \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\hat{\beta}_i^2} \right\}$
46.	Khalaf (2013)	$k_{46} = \frac{(\lambda_{max} + \lambda_{min})\lambda_{max}\hat{\sigma}^2}{2\sum_{i=1}^{p}\lvert\hat{\beta}\rvert_j (n-p)\hat{\sigma}^2 + \lambda_{max}\hat{\beta}_{max}^2}$
47.	Khalaf, Mansson and Shukur (2013)	$k_{47} = max\left(\frac{1}{m_i}\right)$, where $m_i = \sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_i^2)}}$
48.	Khalaf, Mansson and Shukur (2013)	$k_{48} = max\left(\sqrt{m_i}\right)$ where $m_i = \sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_i^2)}}$
49.	Khalaf, Mansson and Shukur (2013)	$k_{49} = \left(\prod_{i=1}^{p} 1/\sqrt{m_i} \right)^{1/p}$
50.	Khalaf, Mansson and Shukur (2013)	$k_{50} = \left(\prod_{i=1}^{p} \sqrt{m_i} \right)^{1/p}$
51.	Khalaf, Mansson and Shukur (2013)	$k_{51} = median\left(\frac{1}{\sqrt{m_i}}\right)$ where $m_i = \sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_i^2)}}$
52.	Asar et al. (2014)	$k_{52} = \frac{p^2\hat{\sigma}^2}{\lambda_{max}^2 \sum_{i=1}^{p}\hat{\alpha}_i^2}$
53.	Asar et al. (2014)	$k_{53} = \frac{p^3\hat{\sigma}^2}{\lambda_{max}^3 \sum_{i=1}^{p}\hat{\alpha}_i^2}$

54.	Asar et al. (2014)	$k_{54} = \frac{p\hat{\sigma}^2}{\lambda_{max}^{1/3} \sum_{i=1}^{p} \hat{\alpha}_i^2}$
55.	Asar et al. (2014)	$k_{55} = \frac{p\hat{\sigma}^2}{\left(\sum_{i=1}^{p} \sqrt{\lambda_i}\right)^{1/3} \sum_{i=1}^{p} \hat{\alpha}_i^2}$
56.	Asar et al. (2014)	$k_{56} = \frac{2p\hat{\sigma}^2}{\sqrt{\lambda_i} \sum_{i=1}^{p} \hat{\alpha}_i^2}$
57.	Khalaf and Iguernane (2014)	$k_{57} = \frac{\hat{\sigma}^2}{2}\left(\frac{1}{\hat{\alpha}_{max}^2} + \frac{p}{\sum_{i=1}^{p} \hat{\alpha}_i^2}\right)$
58.	Khalaf and Iguernane (2014)	$k_{58} = \hat{\sigma}^2 \sqrt{\frac{p}{\hat{\alpha}_{max}^2 \sum_{i=1}^{p} \hat{\alpha}_i^2}}$
59.	Dorugade (2014)	$k_{59} = \frac{2\hat{\sigma}^2}{\lambda_{max}\hat{\alpha}_i^2}$
60.	Dorugade (2014)	$k_{60} = \frac{2\hat{\sigma}^2}{p\lambda_{max}} \sum_{i=1}^{p} \frac{1}{\hat{\alpha}_i^2}$
61.	Dorugade (2014)	$k_{61} = \frac{2p\hat{\sigma}^2}{\lambda_{max} \sum_{i=1}^{p} \hat{\alpha}_i^2}$
62.	Doruge (2014)	$k_{62} = Median\left(\frac{2\hat{\sigma}^2}{\lambda_{max}\, \hat{\alpha}_i^2}\right)$
63.	Doruge (2014)	$k_{63} = \frac{2\hat{\sigma}^2}{\lambda_{max} \left(\prod_{i=1}^{p} \hat{\alpha}_i^2\right)^{1/p}}$
64.	Göktaş and Sevinç (2016)	$k_{64} = \sqrt{median\left(\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}\right)}$
65.	Göktaş and Sevinç (2016)	$k_{65} = \frac{\hat{\sigma}^2}{(median(\hat{\alpha}_i))^2}$
66.	Khalaf and Iguern (2016)	$k_{66} = \sqrt{\frac{\lambda_{max}\, \hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2 + \lambda_{max}\, \hat{\beta}_{max}^2)}}$
67.	Bhat and Vidya(2016)	$k_{67} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2} + \frac{1}{\lambda_{max} \sum_{i=1}^{p} \hat{\alpha}_i^2}$
68.	Bhat and Vidya(2016)	$k_{68} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2} + \frac{1}{2\sqrt{\lambda_{max}/\lambda_{min}}}$
69.	Karaibrahimoglu, *et al.* (2016)	$k_{69} = \frac{\sqrt{5}p\hat{\sigma}^2}{\lambda_{max} \sum_{i=1}^{p} \hat{\alpha}_i^2}$
70.	Karaibrahimoglu, *et al.* (2016)	$k_{70} = \frac{p\hat{\sigma}^2}{\sqrt{\lambda_{max}} \sum_{i=1}^{p} \hat{\alpha}_i^2}$

71.	Karaibrahimoglu, *et al.* (2016)	$k_{71} = \frac{2p\hat{\sigma}^2}{\sum_{i=1}^{p} \lambda_i^{1/4} \sum_{i=1}^{p} \hat{\alpha}_i^2}$
72.	Karaibrahimoglu, *et al.* (2016)	$k_{72} = \frac{2p\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p} \lambda_i} \sum_{i=1}^{p} \hat{\alpha}_i^2}$
73.	Dorugade (2016)	$k_{73} = \hat{\sigma}$, where $\hat{\sigma} = \sqrt{\sum_{j=1}^{n} \hat{e}_j / (n-p)}$.
74.	Lukman et al. (2017)	$k_{74} = \sqrt{\frac{2\hat{\sigma}^2}{\lambda_{max} max(\hat{\alpha}_i^2)}}$
75.	Lukman et al. (2017)	$k_{75} = max\left(1 \Big/ \left(\frac{2\hat{\sigma}^2}{\lambda_{max} \hat{\alpha}_i^2} \right) \right)$
76.	Lukman et al. (2017)	$k_{76} = \frac{2\hat{\sigma}^2}{\lambda_{max}} \frac{1}{p} \sum_{i=1}^{p} \frac{1}{\hat{\alpha}_i^2}$
77.	Lukman et al. (2017)	$k_{77} = \frac{2p\hat{\sigma}^2}{\lambda_{max} \sum_{i=1}^{p} \hat{\alpha}_i^2}$
78.	Lukman et al. (2017)	$k_{78} = \frac{2\hat{\sigma}^2}{\lambda_{max} \left(\prod_{i=1}^{p} \hat{\alpha}_i^2 \right)^{1/2}}$
79.	Lukman et al. (2017)	$k_{79} = median\left(\frac{2\hat{\sigma}^2}{\lambda_{max} (\hat{\alpha}_i^2)} \right)$
80.	Lukman et al. (2017)	$k_{80} = \frac{2\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p} \lambda_i} max(\hat{\alpha}_i^2)}$
81.	Lukman et al. (2017)	$k_{81} = max\left(\frac{2\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p} \lambda_i} (\hat{\alpha}_i^2)} \right)$
82.	Lukman et al. (2017)	$k_{82} = \frac{2\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p} \lambda_i}} \sum_{i=1}^{p} \frac{1}{\hat{\alpha}_i^2}$
83.	Lukman et al. (2017)	$k_{83} = \frac{2\widehat{p\sigma}^2}{\sqrt{\sum_{i=1}^{p} \lambda_i} \sum_{i=1}^{p} \hat{\alpha}_i^2}$
84.	Lukman et al. (2017)	$k_{84} = \frac{2\widehat{p\sigma}^2}{\sqrt{\sum_{i=1}^{p} \lambda_i} \left(\prod_{i=1}^{p} \hat{\alpha}_i^2 \right)^{1/p}}$

85.	Lukman et al. (2017)	$k_{85} = median\left(\frac{2\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p} \lambda_i \hat{\alpha}_i^2}}\right)$
86.	Asar and Genc (2017)	$k_{86} = \frac{1}{p}\sum_{i=1}^{p}\sqrt{\frac{\hat{\sigma}^2}{\lambda_i \hat{\alpha}_i^2}}$
87.	Asar and Genc (2017)	$k_{87} = \left(\prod_{i=1}^{p}\sqrt{\frac{\hat{\sigma}^2}{\lambda_i \hat{\alpha}_i^2}}\right)^{1/p}$
88.	Asar and Genc (2017)	$k_{88} = median\left(\sqrt{\frac{\hat{\sigma}^2}{\lambda_i \hat{\alpha}_i^2}}\right)$
89.	Asar and Genc (2017)	$k_{89} = max\left(\sqrt{\frac{\hat{\sigma}^2}{\lambda_i \hat{\alpha}_i^2}}\right)$
90.	Asar and Genc (2017)	$k_{90} = median\left(1/\sqrt{\frac{\hat{\sigma}^2}{\lambda_i \hat{\alpha}_i^2}}\right)$
91.	Asar and Genc (2017)	$k_{91} = max\left(1/\sqrt{\frac{\hat{\sigma}^2}{\lambda_i \hat{\alpha}_i^2}}\right)$
92.	Asar and Genc (2017)	$k_{92} = \frac{1}{p}\sum_{i=1}^{p}\left(1/\sqrt{\frac{\hat{\sigma}^2}{\lambda_i \hat{\alpha}_i^2}}\right)$
93.	Asar and Genc (2017)	$k_{93} = p/\sum_{i=1}^{p}\sqrt{\frac{\lambda_i \hat{\alpha}_i^2}{\hat{\sigma}^2}}$
94.	Lukman and Olatunji (2018)	$k_{95} = p\hat{\sigma}$
95.	Ertas (2018)	$k_{94} = \frac{p}{\sum_{i=1}^{p}\frac{\hat{\alpha}_i^2}{\sqrt{\lambda_i \hat{\alpha}_i^2 \hat{\sigma}^2 + \hat{\sigma}^4} + \hat{\sigma}^2}}$
96.	Ertas (2018)	$k_{96} = \frac{1}{p}\sum_{i=1}^{p}\frac{\sqrt{\lambda_i \hat{\alpha}_i^2 \hat{\sigma}^2 + \hat{\sigma}^4} + \hat{\sigma}^2}{\hat{\alpha}_i^2}$
97.	Suhail and Chand (2019)	$k_{97} = \{\hat{k}_{(j)}\}$, such that $P(\hat{k} < KQ_\gamma) = \gamma\{\hat{k}_{(j)}\}$
98.	Suhail and Chand (2019)	$k_{98} = \frac{\sqrt{\lambda_{max}(\hat{\alpha}_i^2)_{0.50}\hat{\sigma}^2 + \hat{\sigma}^4} + \hat{\sigma}^2}{(\hat{\alpha}_i^2)_{0.50}}$ $(\hat{\alpha}_i^2)_{0.50}$ implied 50th percentile of $(\hat{\alpha}_i^2)$

99.	Suhail et al. (2019)	$k_{99} = \frac{\sqrt{\lambda_{max}(\hat{\alpha}_i^2)_{0.75}\hat{\sigma}^2 + \hat{\sigma}^4 + \hat{\sigma}^2}}{(\hat{\alpha}_i^2)_{0.75}}$, $(\hat{\alpha}_i^2)_{0.75}$ implied 75th percentile of $(\hat{\alpha}_i^2)$
100.	Zubair and Adenomon (2021)	$k_{100} = min\left(\frac{\hat{\sigma}^2}{2\hat{\beta}_i^2 + \frac{\hat{\sigma}^2}{\lambda_i}}\right)$
101.	Dar et al. (2022)	$k_{101} = \hat{\sigma}^2 p^m \sqrt{\frac{\lambda_{max}}{\lambda_{min}}}$, where m is an arbitrary constant, λ_{max} and λ_{min} are the largest and the smallest eigenvalues of $X'X$ matrix respectively.
102.	Karakoca (2022)	$k_{102} = \text{argmin}_{k>0}\left\{(n-p)\hat{\sigma}^2 \sum_{i=1}^{p}\left(\frac{\lambda_i}{(\lambda_i + k)^2}\right) + k^2 \sum_{i=1}^{p}\left(\frac{\hat{\alpha}_i^2}{(\lambda_i + k)^2}\right) + \frac{k^2}{\hat{\sigma}^2}\right\}$
103.	Khalf (2022)	$k_{103} = \frac{\hat{\sigma}^2}{\hat{\alpha}_{max}^2}\sqrt{\frac{2}{(\lambda_{max} + \lambda_{min})}}$
104.	Kibria (2022, proposed)	$k_{104} = GM\left(\frac{\hat{\sigma}^2}{\hat{\beta}_i^2} + \frac{1}{\lambda_i}\right)$, where GM stands for geometric mean.
105.	Kibria (2022, proposed)	$k_{105} = HM\left(\frac{\hat{\sigma}^2}{\hat{\beta}_i^2} + \frac{1}{\lambda_i}\right)$, where HM stands for harmonic mean.
106.	Kibria (2022, proposed)	$k_{106} = Median\left(\frac{\hat{\sigma}^2}{\hat{\beta}_i^2} + \frac{1}{\lambda_i}\right)$
107.	Kibria (2022, proposed)	$k_{107} = max\left(\frac{\hat{\sigma}^2}{\hat{\beta}_i^2} + \frac{1}{\lambda_i}\right)$ where max stands for maximum.
108.	Kibria (2022, proposed)	$k_{108} = min\left(\frac{\hat{\sigma}^2}{\hat{\beta}_i^2} + \frac{1}{\lambda_i}\right)$ where min stands for minimum.

4. Some Concluding Remarks

This paper considers more than 100 different estimators for the shrinkage parameter of the ridge regression models, which have been published since 1970 (Hoerl and Kennard 1970), when the errors of the model are normally distributed. These estimators may be used for the other ridge regression models, such as, Poisson, Zero inflated Poisson, Gamma, Logistic, Negative binomial, Zero inflated negative Binomial, Bell, inverse Gaussian regression, semi-parametric regression models among others. This paper made an attempt to review most possible published papers until now on the estimation of ridge or shrinkage parameter k for the linear regression model. Since the choice of k depends on the particular sample under investigation and may vary from sample to sample, the properties associated with ridge regression for fixed *k* may not hold. We just reported more than 100 different shrinkage estimators. Different researchers have proposed different estimators at different times and compared their performances under different simulation conditions, which is not comparable as a whole. Therefore, it will be interesting if we compare them under the same simulation conditions and proposed some good estimators for the practitioners. Such possibility is under consideration in a separate paper. We expect that this paper will bring a lot of attention among the researchers and will be a reference paper in the area of ridge regression.

Dedications: B M Golam Kibria wises to dedicate this paper to Bangabandhu Sheikh Mujibur Rahman, Father of the Nation, for his invaluable sacrifice, great leadership, and the commitment for the independence of Bangladesh.

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